



Mathematical Concepts and Emergent Numeracy in Early Years

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Structure of the Session

How Early
Mathematics Support
Cognitive Development
In Early Years

Early Mathematical Skills
And Milestones

Emergent Mathematics
And Numeracy: The
Concept

Mathematical And
Numerical Competencies

Ways To Support
Emergent Mathematics
In Early Years With
Practical Activities

HOW EARLY MATHEMATICS SUPPORT COGNITIVE DEVELOPMENT IN EARLY YEARS

COGNITIVE DEVELOPMENT

- It refers to the development of mental processes and capabilities;
- It focuses on how children learn and process information;
- It is the development of the thinking and organizing systems of the mind;
- It involves language, imagining, thinking, exploring, reasoning, problem solving, developing and rejecting ideas and concepts, memory, expression through multiple media, experimenting and applying what they learn.

Creative Thinking

Creative Thinking Helps in:

- Problem solving
- Different way to do things
- Idea generation
- Use of materials innovatively
- Risk taking
- Experimenting
- Making mistakes and learning

Creative Thinking Activities:

- Imagine all the different ways to get to school (walking, flying, driving, swimming).
- Name everything he/she can think of that is red.
- Name everything he/she can think of that's round.
- Imagine all the things he/she could make out of clay or paper bags or even an empty box.



Critical Thinking

Critical, or logical, thinking is the ability to break an idea into its parts and analyze them.

Cognitive Activities:

- How many different ways he/she can sort his blocks in?
- How many different ways he/she can make a building out of the blocks in?
- How the building would be different if he/she used blocks of only one size?
- How a bottle of juice and his/her lunch box are alike and how are they different?
- How family members' shoes are alike and how are they different?



EARLY MATHEMATICAL SKILLS AND MILESTONES

Mathematical Skills

- Math skills help individuals deal with basic, everyday tasks —from getting to work on time to paying bills.
- They are the practical abilities that are useful no matter the industry or size of business you work in.
- Mathematical skills are conceptualized as a separate area that includes verbal components (number knowledge, counting, computation, and reasoning) and nonverbal components (math notation, reasoning in time and space, and computation).



Mathematical Milestones from 0 till 8 years

Babies (ages 0–12 months)

- Begin to predict the sequence of events (like running water means bath time)
- Start to understand basic cause and effect (shaking a rattle makes noise)
- Begin to classify things in simple ways (some toys make noise and others don't)
- Start to understand relative size (baby is small, parents are big)
- Begin to understand words that describe quantities (*more, bigger, enough*)



Toddlers (ages 1–2 years)

- Understand that numbers mean “how many” (using fingers to show how many years old they are)
- Begin reciting numbers, but may skip some of them
- Understand words that compare or measure things (*under, behind, faster*)
- Match basic shapes (triangle to triangle, circle to circle)
- Explore measurement by filling and emptying containers
- Start seeing patterns in daily routines and in things like floor tiles



Preschoolers (ages 3–4 years)

- Recognize shapes in the real world
- Start sorting things by color, shape, size, or purpose
- Compare and contrast using classifications like height, size, or gender
- Count up to at least 20 and accurately point to and count items in a group
- Understand that numerals stand for number names (5 stands for *five*)
- Use spatial awareness to put puzzles together
- Start predicting cause and effect (like what will happen if they drop a toy in a tub full of water)



Kindergartners (age 5 years)

- Add by counting the fingers on one hand — 1, 2, 3, 4, 5 — and starting with 6 on the second hand
- Identify the larger of two numbers and recognize numerals up to 20
- Copy or draw symmetrical shapes
- Start using very basic maps to find a “hidden treasure”
- Begin to understand basic time concepts, like morning or days of the week
- Follow multi-step directions that use words like first and next
- Understand the meaning of words like unlikely or possible



First and Second Graders

- Predict what comes next in a pattern and create own patterns
- Know the difference between two- and three-dimensional shapes and name the basic ones (cubes, cones, cylinders)
- Count to 100 by ones, twos, fives, and tens
- Write and recognize the numerals 0 to 100, and the words for numbers from one to twenty
- Do basic addition and subtraction up to 20
- Read and create a simple bar graph
- Recognise and know the value of coins



Third Graders

- Move from using hands-on methods to using paper and pencil to work out math problems
- Work with money
- Do addition and subtraction with regrouping (also known as borrowing)
- Understand place value well enough to solve problems with decimal points
- Know how to do multiplication and division, with help from fact families (collections of related math facts, like $3 \times 4 = 12$ and $4 \times 3 = 12$)
- Create a number sentence or equation from a word problem



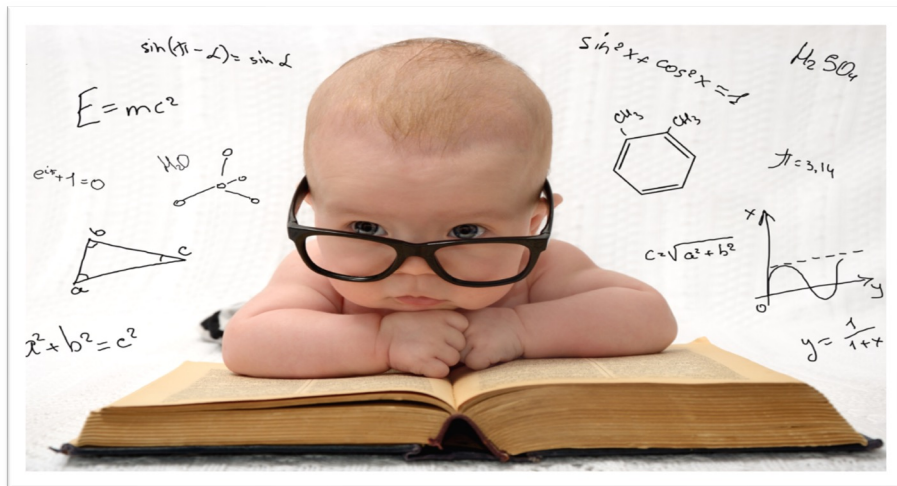
EMERGENT MATHEMATICS AND NUMERACY: THE CONCEPT

Early Mathematics and Numeracy Skills (Video)



What is Emergent Mathematics?

Emergent mathematics is a term to describe how children construct **mathematics** from birth and continuing throughout the life of the person through a combination of cognitive development and interaction with their environment.



Numeracy Stages

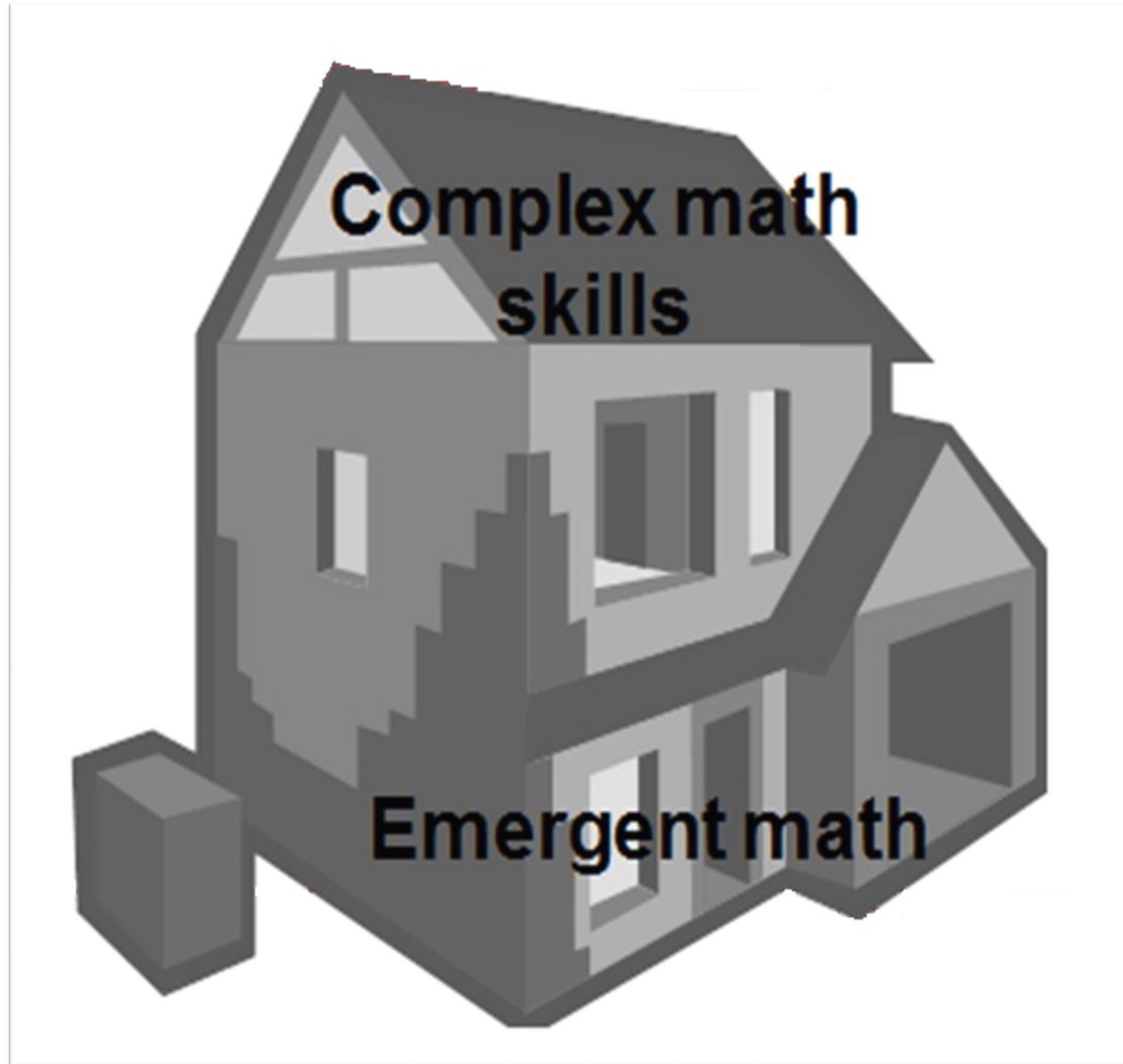


- 0: Emergent
- 1: One to One Counting
- 2: Count from one on Materials
- 3: Count from one by Imaging
- 4: Advanced Counting
- 5: Early Additive Part-Whole
- 6: Advanced Additive Part-Whole
- 7: Advanced Multiplicative
- 8: Advanced Proportional

Counting
Strategies

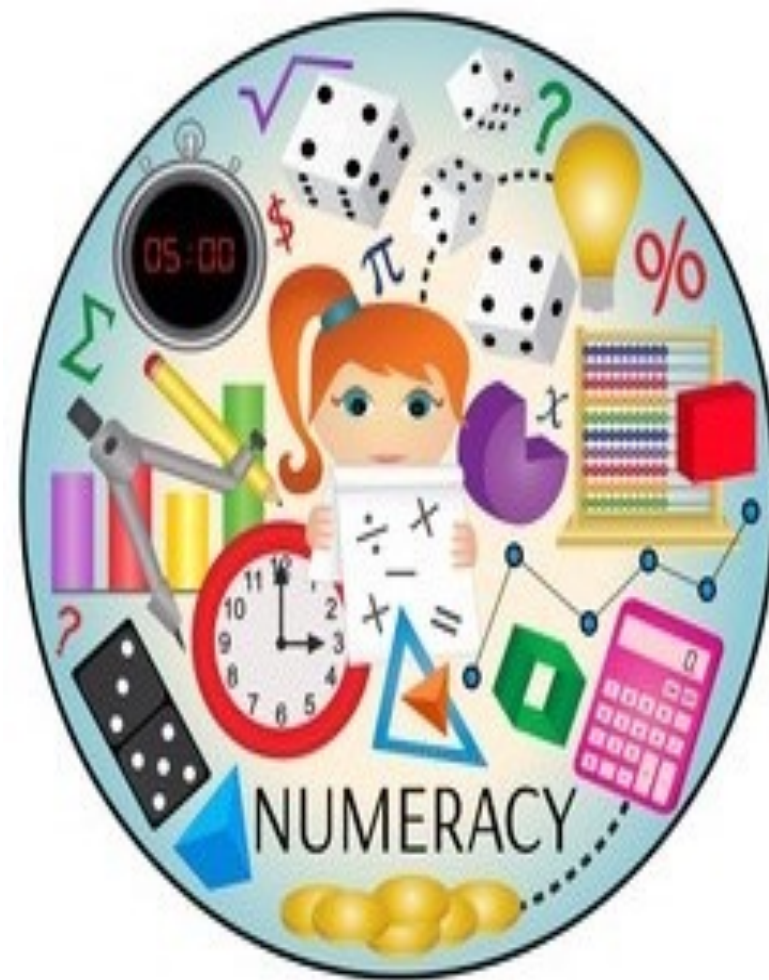
Non Counting
Strategies

What is Emergent Mathematics?



What is Numeracy?

Numeracy is the ability to reason and to apply simple numerical concepts. Basic numeracy skills consist of comprehending fundamental arithmetical operations like addition, subtraction, multiplication, and division.



Numeracy (Cont....)

- Children start learning **numeracy** skills from the time they are born. This learning happens through everyday play and activities – for example, **when** you encourage your child to: count fingers, toes and toys, recognize numbers on objects like clocks or remote controls.
- A child's early years are a time of rapid learning. Research tells us that babies have an innate capacity to understand numbers. As your child's first teacher, you play a key role in developing their numeracy skills from an early age.



Importance of Emergent Numeracy Skills

- Developing early numeracy skills gives children a foundation for their learning and development.
- It prepares them for daily life, including general problem solving and handling money.
- Math includes noticing numbers, shapes, patterns, size, time and measurement.



How Early Numeracy Develops

- Children are using early math skills throughout their daily routines and activities.
- Even before they start school, most children develop an understanding of addition and subtraction through everyday interactions.
- Other math skills are introduced through daily routines shared with the child—counting steps as go up or down.



MATHEMATICAL AND NUMERICAL COMPETENCIES

Numbers and Counting

Children's understanding of numbers, quantity, order, ways of representing numbers, one-to-one correspondence, and counting.

Associated Skills:

- Child can count to ten or higher.
- Child can identify and name the numbers.
- Child can tell if there is more or fewer when an object is added or removed from a small group of objects.
- Child can tell the difference between “some” and “all.”
- Child can use number words to describe “how many” (“four sisters”)
- Child can match up numerals (1,2,3 etc.) to sets of objects (three leaves)

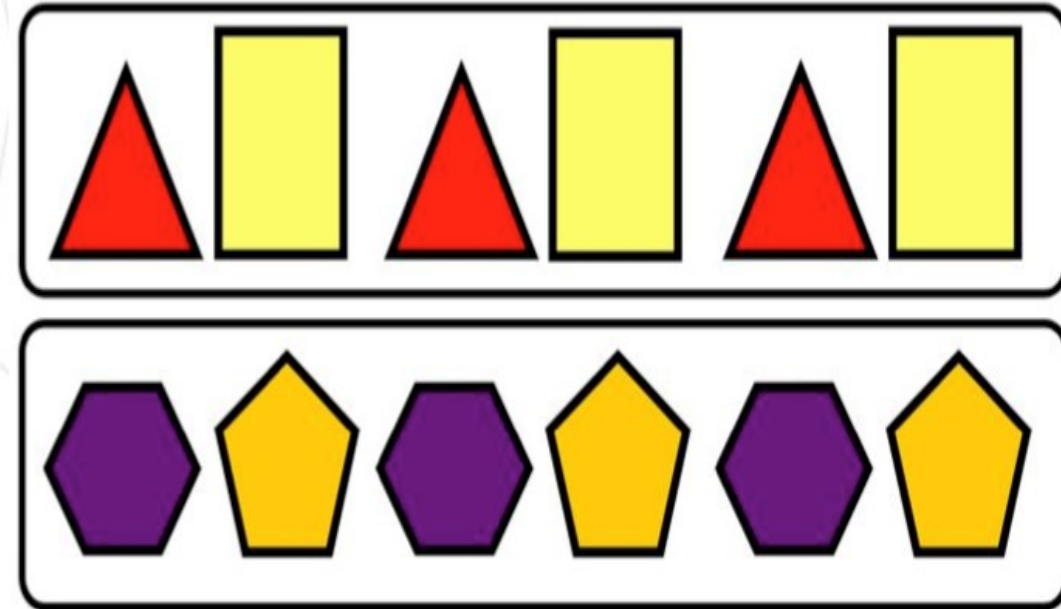


Patterns

Children's ability to understand patterns, which are things that repeat.

Associated Skills:

- Child can recognize patterns—starting with simple to more complex ones.
- Child can copy, extend and later generate his own patterns—starting with simple to more complex ones.

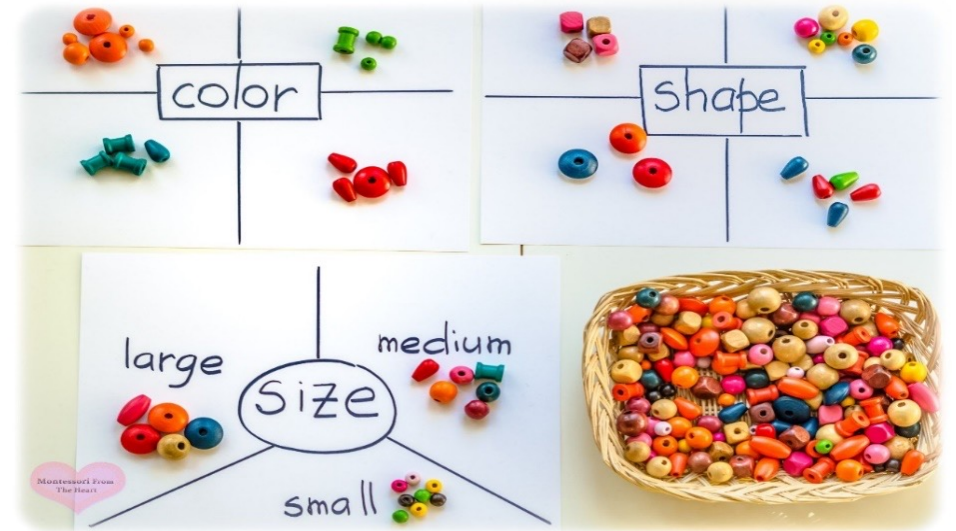


Sorting and Classification

Children's ability to distinguish between alike and different objects and to sort them according to one or more attributes.

Associated Skills:

- Child can sort and organize objects using one and later more than one characteristic.
- Child can match alike objects.
- Child can arrange numbers and objects in a series (e.g. big to small).

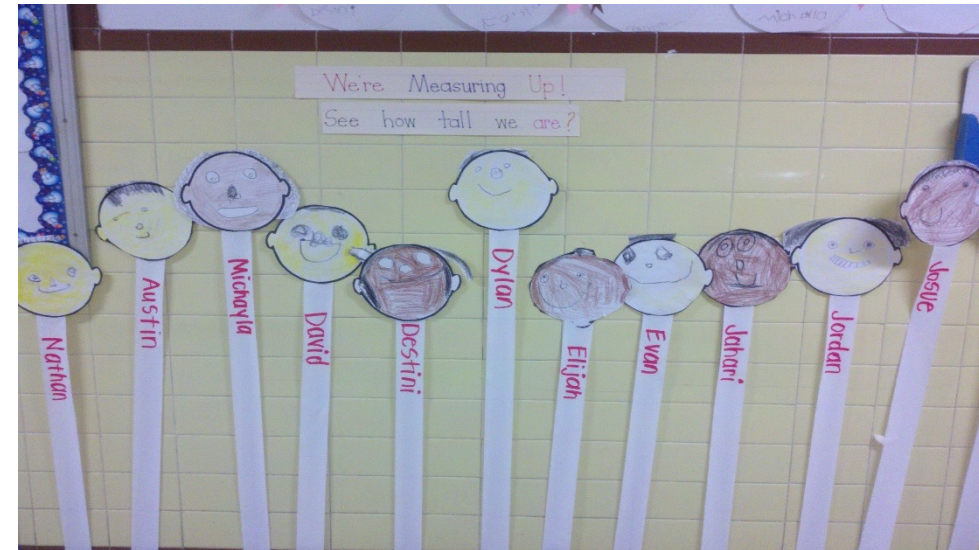
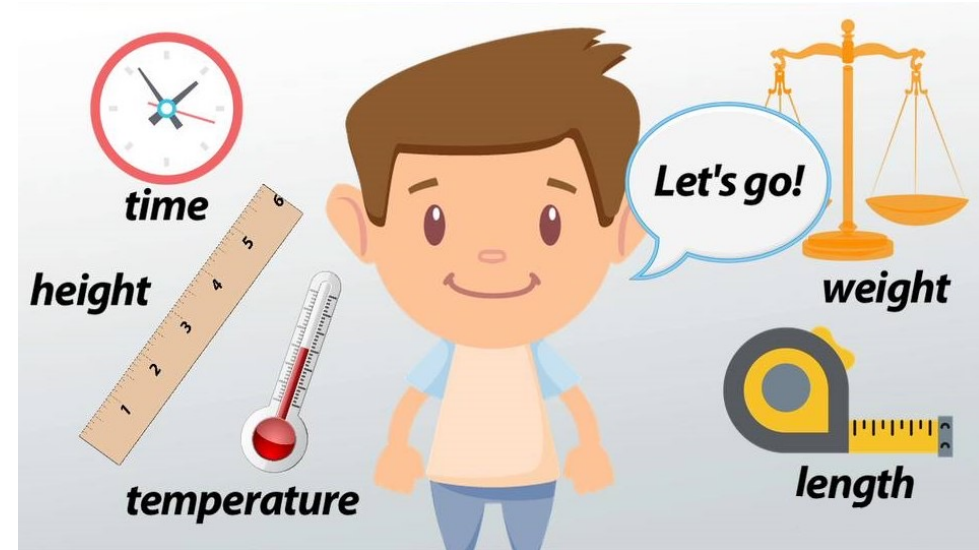


Comparison and Measurement

Children's ability to describe and compare measurable attributes, as well as to classify objects by certain attributes.

Associated Skills:

- Child can remember and talk about things that have happened.
- Child can talk about “before” and “after.”
- Child can decide that something is bigger than something else.
- Child can decide that something is heavier than something else.

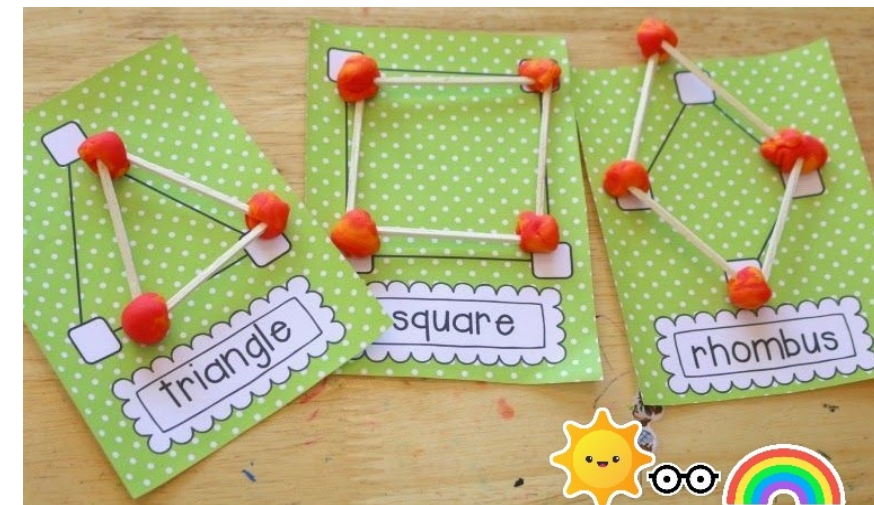


Geometry

This knowledge area refers to a child's ability to understand ideas about shapes, size and position.

Associated Skills:

- Child can use and identify shapes (circle, square, triangle, etc.).
- Child can throw and catch objects.
- Child can stack things up or move things around in space.



WAYS TO SUPPORT EMERGENT MATHEMATICS IN EARLY YEARS WITH PRACTICAL ACTIVITIES

The Everyday Mathematics Experiences in Early Years

- Children are using early math skills throughout their daily routines and activities.
- Early mathematical concepts and skills that first-grade mathematics curriculum builds on include: (Bowman et al., 2001, p. 76):
 1. Understanding size, shape, and patterns
 2. Ability to count verbally (first forward, then backward)
 3. Recognizing numerals
 4. Identifying more and less of a quantity
 5. Understanding one-to-one correspondence (i.e., matching sets, or knowing which group has four and which has five)

Incorporating Mathematics:

Ideas for talking

- Use Mathematics concepts to describe what parent and child are seeing and doing together. For example, 'Look at the fast cars' or 'This bag is heavy'.
- When parent is preparing food, she/he can talk about what is being done. For example, 'I'm cutting this orange in half' or 'Let's share these grapes – one for me and one for you'.
- Point out and name the numbers their child sees, like the numbers on mailboxes, buses and road signs.
- When parent and child are out and talk about what is near or further away. For example, 'Let's sit on that bench nearby to have our snack' or 'It's quite far to the lake. Would you like to ride in the stroller?'
- Talk about activities that happen at certain times of the day. For example, 'We eat breakfast at 7 am', or 'Let's go to the park before we have dinner at 6 pm'.

Incorporating Mathematics: Ideas for Everyday Activities

- Make counting part of child's everyday life. For example, count shells at the beach, fruit at the shop and trees on the street. Or count toys together as child packs them away.
- When parent and child are together outdoors, encourage child to describe or compare shapes of leaves, colours of flowers or sizes of birds.
- Child can be taken for a walk down the street and point out how each house or block has a number in a series, guess the number of steps between one house and the next.
- Use a growth chart or marks on a wall to measure child's growing height, and describe to the child what you're doing.
- Involve the child in cooking. The child can help stir, pour, fill and mix. This helps the child get familiar with concepts like counting, measuring, adding and estimating.

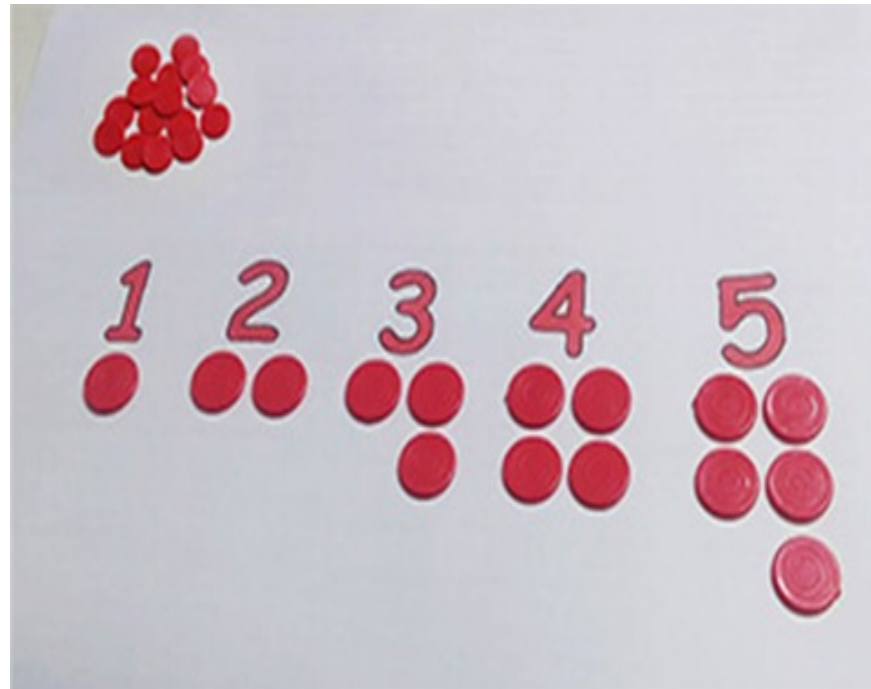
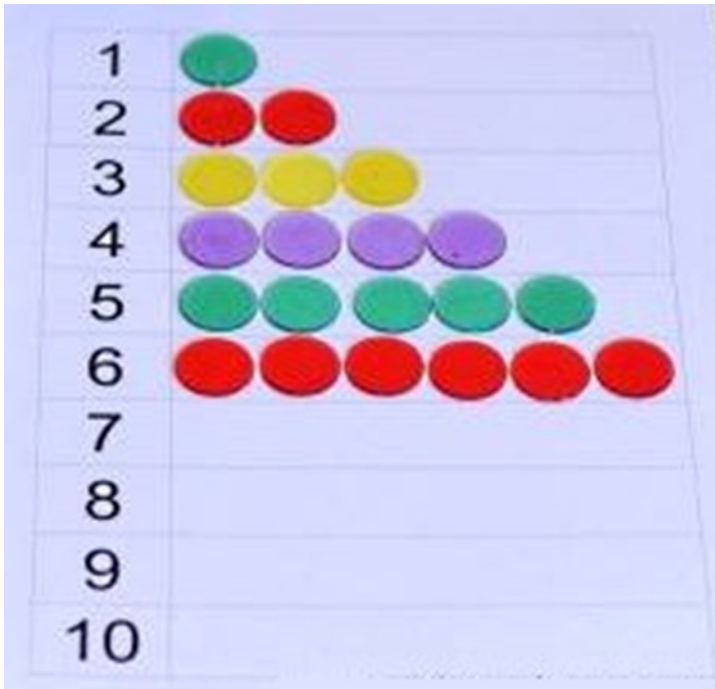
Incorporating Mathematics:

Ideas for play

- Go for a nature walk and let the child gather a mix of leaves, sticks, pebbles and other natural items. The child can sort them into groups based on size, colour, shape or what they do.
- Sing songs and read books with repeating, rhyming or rhythmic numbers. This will help the child understand patterns.
- Play simple board games, card games and puzzles with shapes and numbers, like 'Snap', or matching pairs or dominoes.
- Play outside games like 'I spy', hopscotch, skittles and 'What's the time Mr. Wolf'.
- Play or sing music at different speed. The child can dance, jump or shake musical instruments to slow or fast songs. Sing nursery rhymes slowly and then speed up.
- Race toy cars and talk about which came first, second or third.
- Help the child to arrange their toys in order from shortest to tallest.

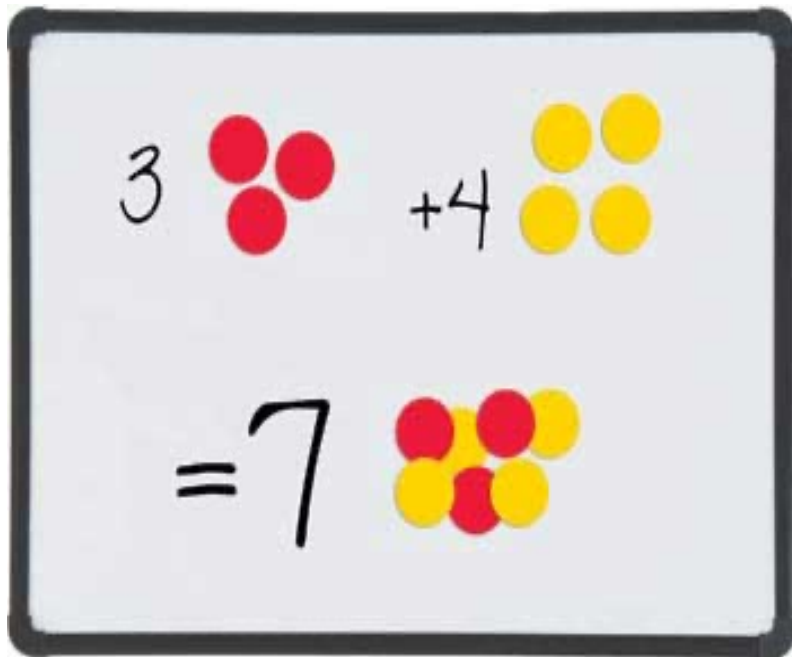
Activities to Enhance Mathematics Skills

- Number poems
- Buttons, straws, pebbles, spoons, etc. for counting, addition, subtraction, multiplication, division.
- Graph (colour, birthday graph)



Activities to Enhance Mathematics Skills

- Finding numbers or mathematical symbols in the Newspaper.
- Telling stories from daily life to connect with mathematics.
- Making a “Balance” using indigenous material, to develop the concept on weighing.



Activities to Enhance Mathematics Skills

- Role plays (organizing a birthday party for eight people, list down the quantity of the things required,) etc.
- Book Making (on numbers, shapes, number stories)
- Story Telling (related to numbers, like 100 days in a school)
- Involve children in games to enhance mental math.
- Paper plate clock and practice time telling



Talking about Mathematics

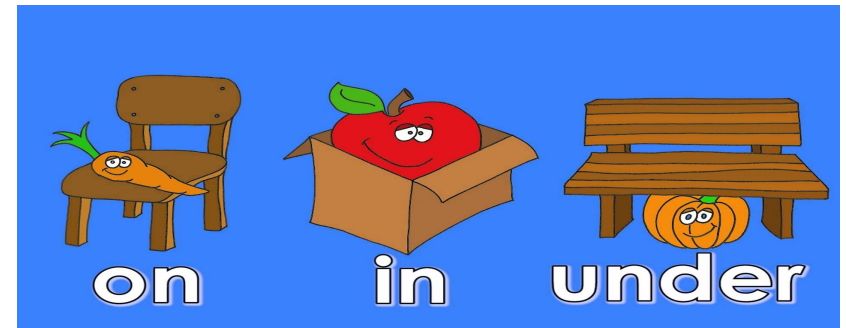
Some activities to develop mathematical language:

- Use specific terms when asking for items. For example, ask your child to get the 'one litre' milk bottle from the fridge.
- When cooking, discuss different measurements used, such as teaspoons, milliliters, litres, and cups. Discuss ideas about empty and full.



Talking about Mathematics

- As you walk, talk and play together describe your child's movements. For example as they climb '**over**' the fence, slide '**between**' the poles, and swing '**under**' the monkey bars. This helps your child understand language related to spatial awareness.
- Sorting activities support your child to understand concepts such as '**same**' and '**different**'. Use recycling as an opportunity to sort items to place in the rubbish. For example, paper, plastic, food waste and general waste.



Counting

- Listen for the counting sequence in these songs and rhymes, which are all on YouTube:

Five Little Ducks

1, 2, 3, 4, 5, Once I Caught a Fish Alive

Ten Green Bottles

Five Little Monkeys

1, 2, Buckle My Shoe



- Children will begin by counting all objects in a group. For example **fingers and toes, the buttons on their clothes, steps to the house, or their toys.** As children move on to counting a set of objects, they begin to link each object with one number. In the beginning, encourage your child to touch each object as they say the matching number.

Counting

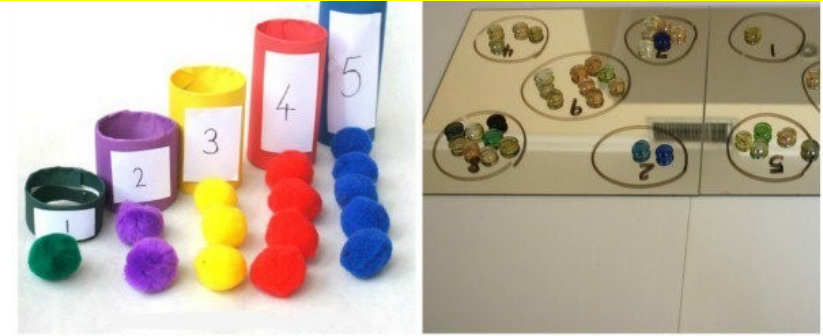
- When beginning to count a group of objects, children may need to arrange the objects in a line to help them count. Later they will be able to start counting from any object without arranging the objects.
- Once your child is confident, use different numbers to start practicing counting. For example, start counting from 6 or 10. Ask your child to count forwards and backwards. Ask what number comes before, or what number comes after, a given number.



Counting

Incorporate counting into everyday activities such as:

- Cut fruit into six pieces and ask your child to count the pieces.
- Count the pieces of toast you cooked at breakfast.
- Add the total number of cutlery items at the table.
- Count the number of people travelling in the car or on the bus.



Counting

- Count the number of houses as you walk along the street.
- Count how many steps it takes to walk from the kitchen to the bathroom.
- Practice counting when grocery shopping with your child. For example, counting the number of apples you put into the bag. Encourage your child to talk about the number of things in the pictures they draw.



Hunting for Numbers

- Number hunts are a fun and engaging activity for your child.
- Ask your child to find numbers around you.
- Say the numbers on houses, car number plates, signs, calendars, newspapers and catalogues.

Using Play Cards

Playing with cards is always a fun activity, particularly on a rainy day or on holidays.

- Play matching number games like 'Snap' with playing cards.
- Order the numbers on the cards from smallest to largest, or largest to smallest.



Playing Shop

Playing shop helps ground your child's math learning while. It also develops their social skills. One way to play shop is to create a mini-shop at home, e.g.

- Collect food and grocery items and label them with prices.
- Talk about how we pay for items using coins, notes and cards.
- Make paper money or use play money to buy and sell goods from the mini-shop.

Playing Shop

- Collect old receipts or price tags and use them in the mini-shop.
- Notice the different shapes, animals and people shown on coins. Discuss the differences. Create coin rubbings with pencils and paper.
- Encourage your child to organize food items by height or by cost.
- Introduce kitchen scales to the mini-shop to weigh foods and organize items by weight.



Playing Games

Making math fun and interactive by playing games will help engage your child. Like:

- Play 'I Spy' or other games to help your child identify shapes, numbers and patterns.
- Board games are a fun way to involve the whole family with math. Help your child when rolling dice to count, move, and stop after moving the number shown on the dice.



Playing Games

- When using dice your child may count all the dots on the die face to determine the total number. Over time they will recognize the value on the die face without counting.
- Play number games online with your child.



Playing with Shapes

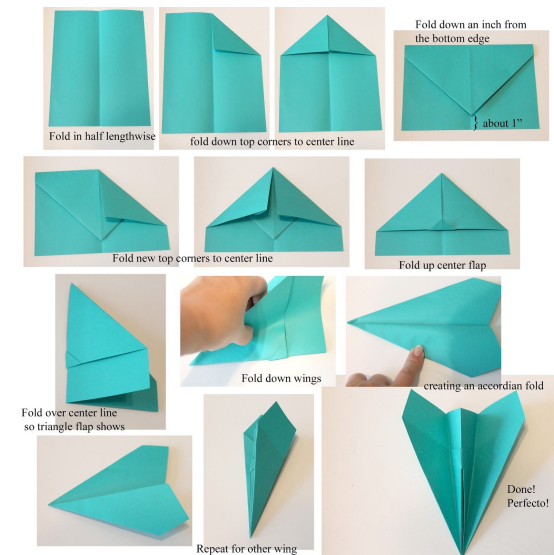
Playing with shapes helps develop your child's awareness of different shapes. It also improves their hand-eye coordination. Like:

- Jigsaw puzzles and shape sorting toys help teach your child problem solving skills and spatial awareness.
- Name and notice the similarities and differences between shapes. For example, shapes with curves, corners or edges.
- Help your child draw shapes, cut them out and sort them into groups. Ask your child to explain why they have sorted the shapes this way.



Playing with Shapes

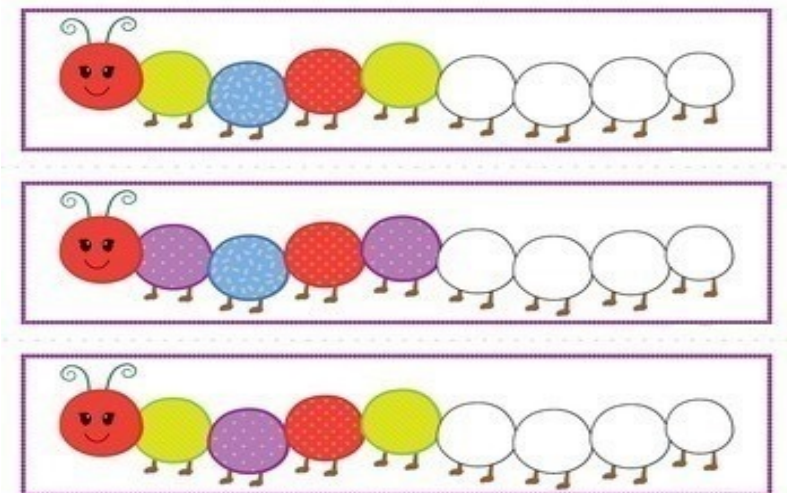
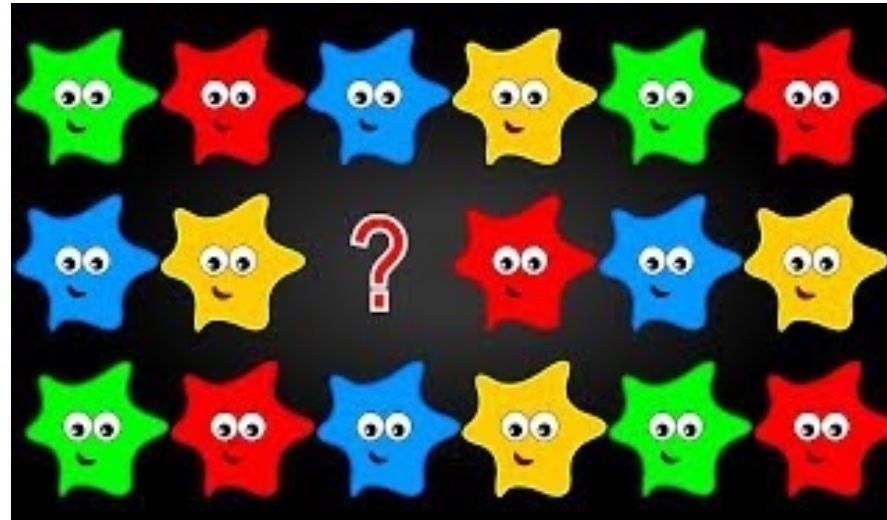
- Use cookie cutters to explore different shapes using play dough. Encourage your child to identify shapes in their everyday life, such as a round ball.
- Make paper planes together. This combines many mathematical concepts, including angles, shapes, halving and symmetry. Once complete, you can compare which plane flew the furthest and have fun measuring too.
- Use building blocks to create a tower. Using the same number of blocks, ask your child to build another tower that's different to the first tower.



Making Patterns

Recognizing and making patterns are important skills for exploring numbers, shapes and symmetry. e.g.

- Identify and explain visual patterns on clothing, wrapping paper, buildings, and furniture. Create a scrapbook for ideas during arts and crafts.
- Use coloured pegs, blocks, beads or cutlery to begin a pattern for your child to continue. Once confident, ask them to create a pattern of their own.



Making Patterns

- Try to incorporate some patterns in rhythm. Create a clapping pattern and ask your child to copy and then create their own pattern.
- Encourage your child to draw, create and describe their own patterns. Use them for borders on greeting cards.



Movement of the Body with Mathematics

These ideas use movement of the body to experience counting:

- Count each toss of the ball as you play a game.
- Estimate how many jumps it will take to get to... Then count how many jumps it takes to get to...
- Count with your child as you climb steps or walk from the park bench to the slide.
- Ask your child to find ways to balance their weight with a friend on the see-saw.
- Sing rhymes and songs that involve counting while skipping.
- Older children can tell how long it will take to reach or fill a glass with big spoon/sponge.

Measuring Things

Understanding measurement and scale are crucial to your child's understanding of math.

- Use a wall measuring chart to measure the height of people in your family.
- Talk to your child about objects around them. Help them judge which is bigger or smaller, taller or shorter.



Measuring Things

- Cut a piece of string for your child – any length will do. Use the string to measure the objects in your house. Find out what is longer or shorter than your 'string measuring tape'. Ask your child to identify anything that is the same length.
- Explore other ways of measuring, such as using footprints or hand lengths.



Measuring Things

- Help your child to build a tower of blocks that is taller than a favourite toy. Ask your child to count the total blocks to measure the height of the tower.
- Estimate and measure who can jump the furthest. Or who can stand on one foot for a longer period, or how many buttons might fill a jar.
- Explore the size of different containers by pouring and filling. Estimate, then check to see which holds more or less.
- Notice changes in the weather and the time of day. Use an old bottle and create a 'rain gauge' to measure and monitor how much it rains.
- Adopt a plant and record its growth by measuring its height.

Asking Questions to Investigate

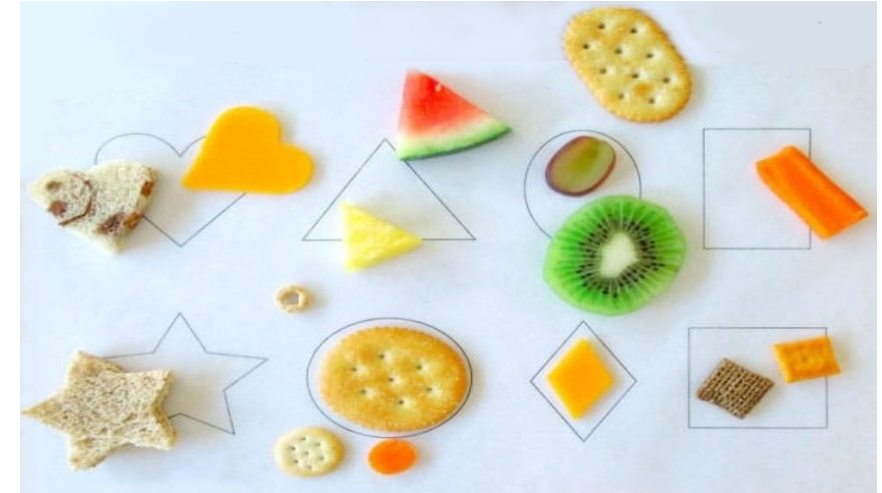
Ask your child questions like these to encourage them to investigate math:

- What shapes can you see?
- How could we measure the...?
- How will we find half?
- What is the best way to share the...?
- How do I get from ... to ...?
- Which is closer: the sandpit or the swing?
- How tall can you build a tower before it falls?



Shapes in the Environment

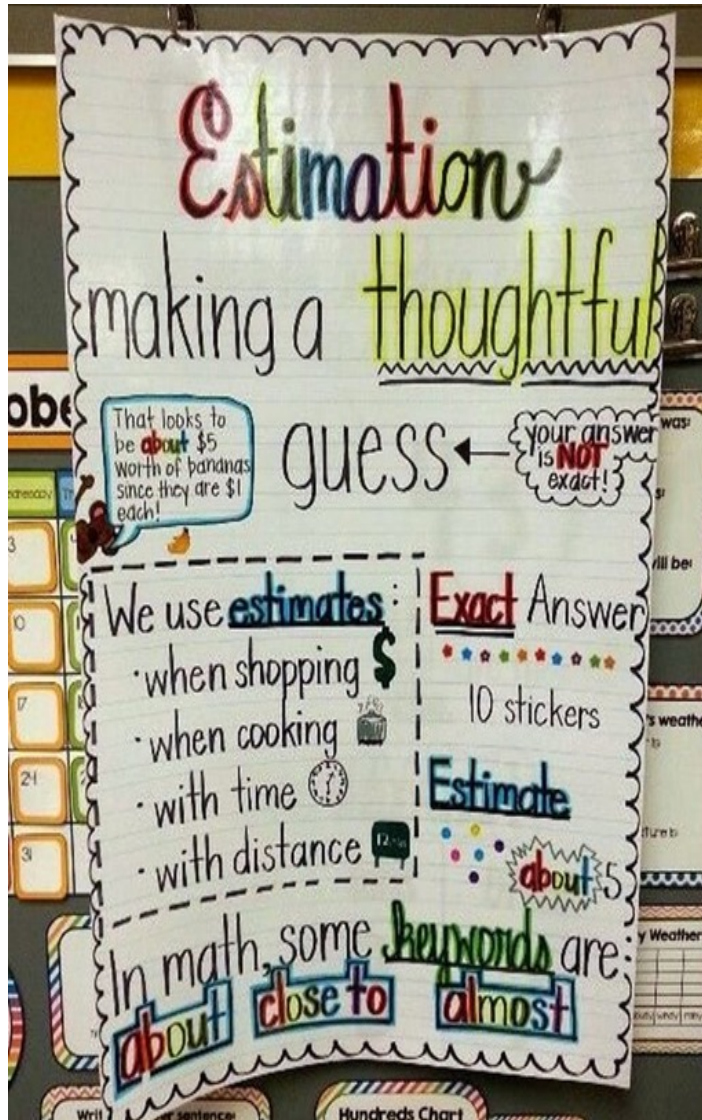
- Play different games to hunt shapes in the environment.
- Encourage children to make new shapes from square, triangle, rectangle and circle.



Calendar Mathematics

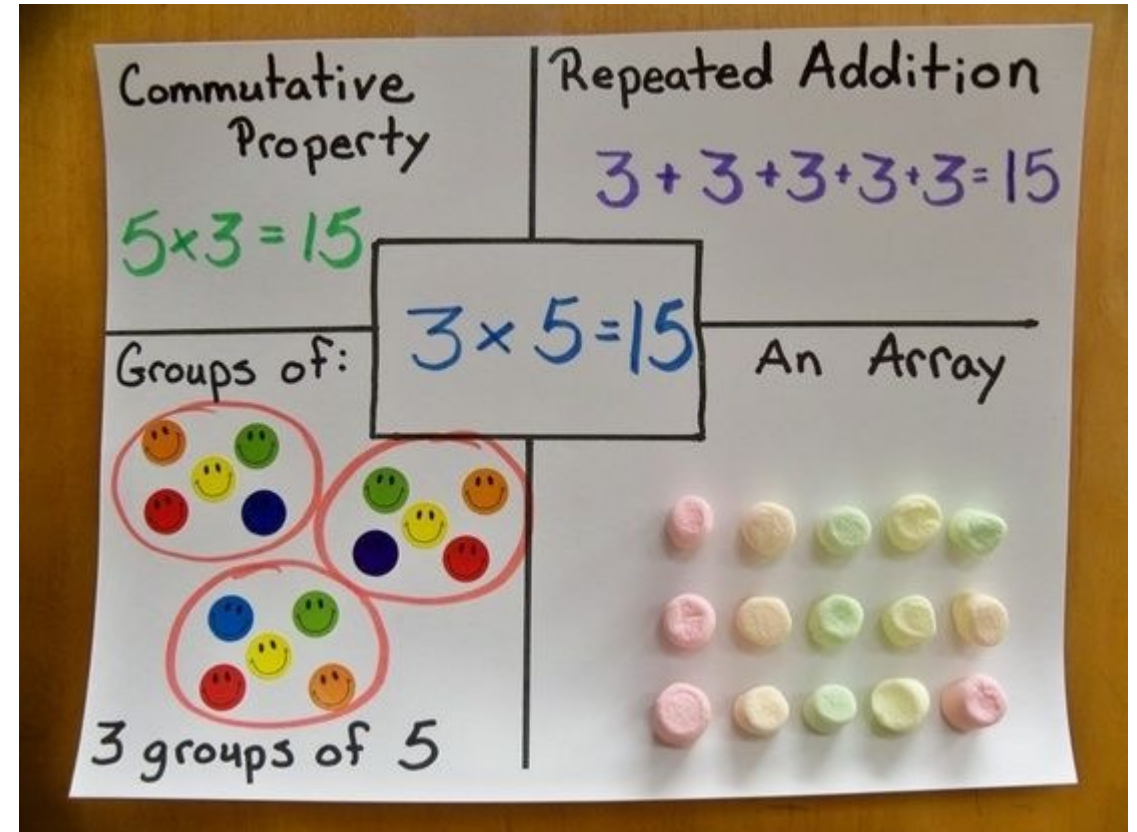


Estimation Game



Concept of Multiplication

- Multiplication is repeated addition
- Learning multiplication by grouping
- Concept of commutative property



Teaching Colours

- Walking rainbow
(https://www.youtube.com/watch?v=fYniUL4I_BA)
- Identify colours in the environment
- I spy with my eye....for colours



Sets

A set is a collection of well-defined objects

Examples from daily life:

1. Kitchen
 2. Shopping malls/shops
 3. School bag
 4. Rules
- <https://www.youtube.com/watch?v=6T16rd15R2E> (sets)
 - <https://www.youtube.com/watch?v=FGlgtywypcw> (classifying into sets)
 - <https://www.youtube.com/watch?v=U4wui1mtotg> (Introducing sets)

Sets



FRACTIONS OF A SET: SPRING THINGS

Let's practice writing fractions!

Please look at the example then complete the rest of the worksheet.

Example:

Here is a set of 3 Easter eggs.

How many Easter eggs are yellow?

1 out of a total of 3 eggs are yellow. We can write this as a fraction: $\frac{1}{3}$

How many Easter eggs are white? $\frac{2}{3}$



Here is a set of 7 jelly beans.

How many jelly beans are blue? $\frac{4}{7}$

How many are white? $\frac{3}{7}$



How many bunnies are there in all? $\frac{4}{4}$ bunnies

How many are white? $\frac{3}{4}$

How many are purple? $\frac{1}{4}$



How many chicks are there in all? $\frac{6}{6}$ chicks

How many are white? $\frac{3}{6}$

How many are green? $\frac{1}{6}$

How many are orange? $\frac{2}{6}$



How many tulips are there in all? $\frac{10}{10}$ tulips

How many are pink? $\frac{6}{10}$

How many are yellow? $\frac{4}{10}$

How many do not have leaves? $\frac{3}{10}$

How many have leaves? $\frac{7}{10}$



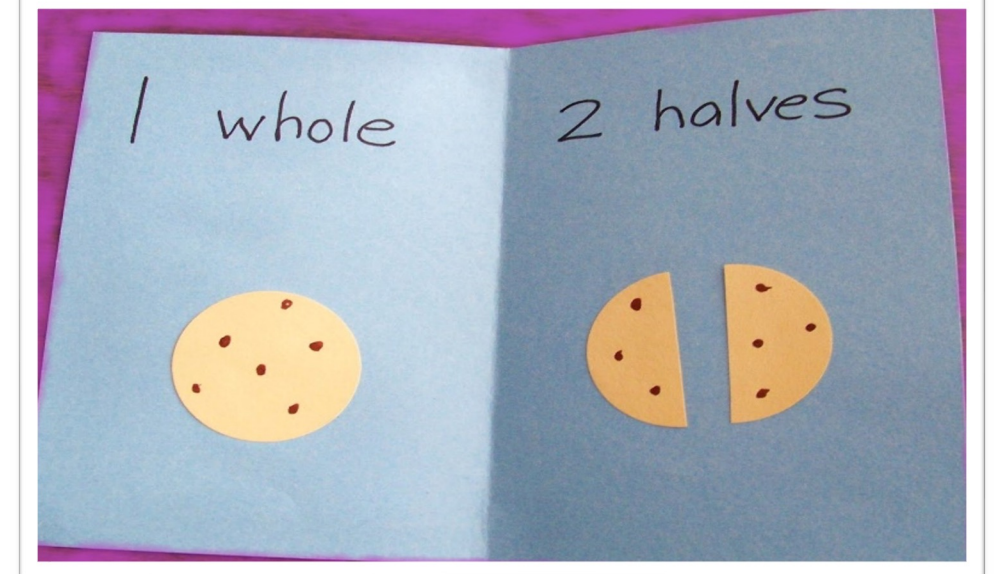
Fractions

Equal part of a whole.

Example from real life:










1. Eating at a restaurant
2. Following recipe
3. Shopping, e.g. 20% off.
4. Marks in a test
5. Time

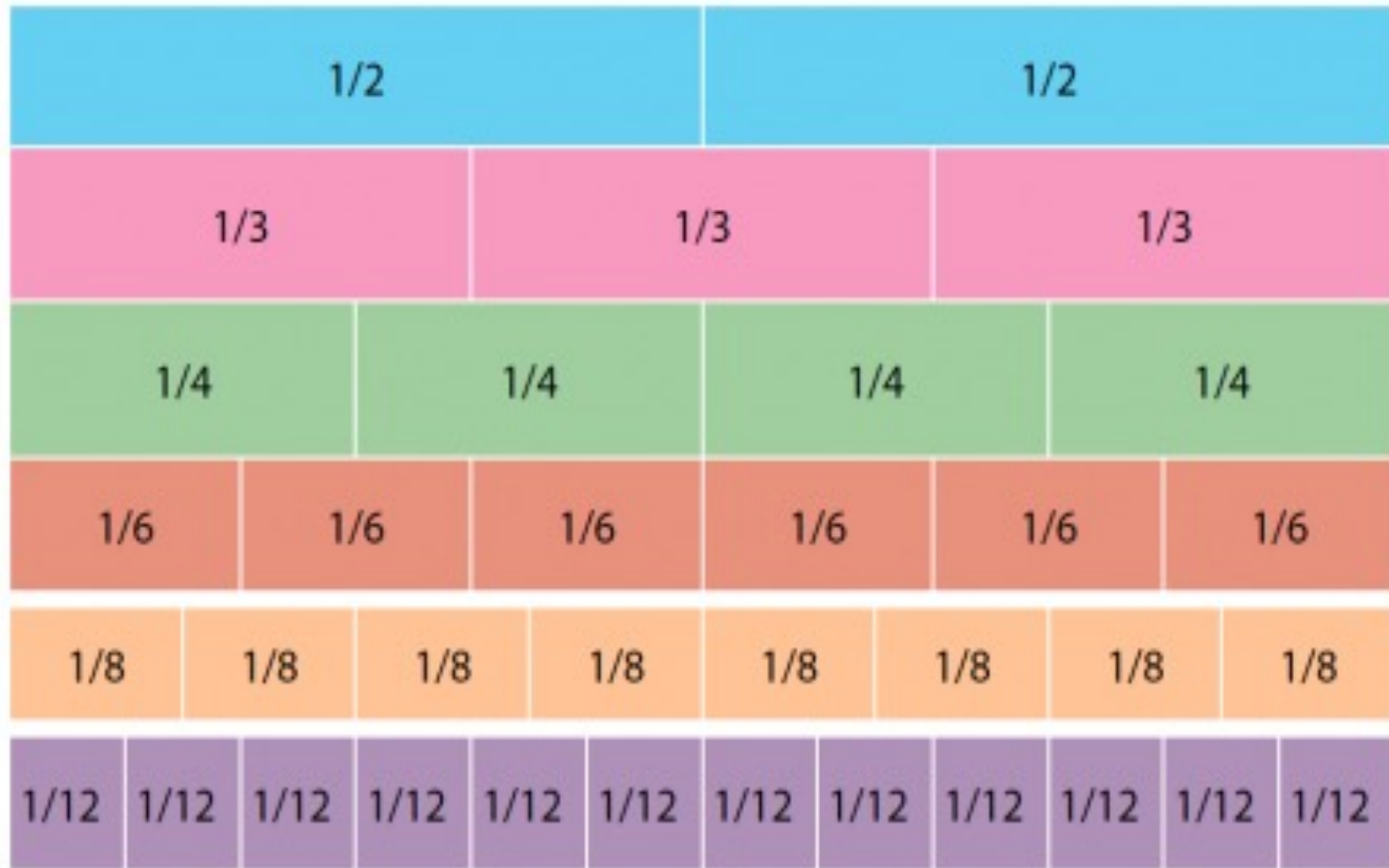
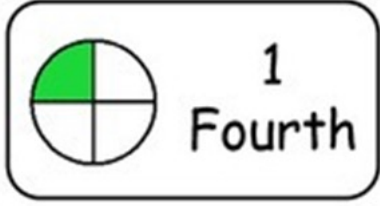
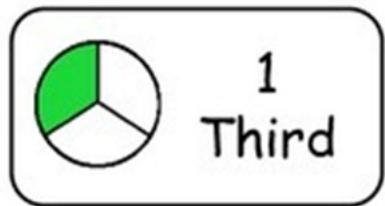
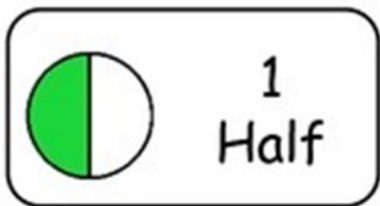
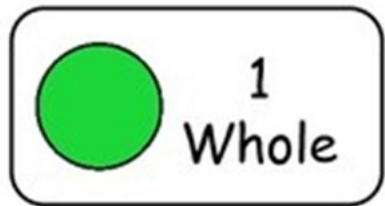
<https://www.youtube.com/watch?v=9j1Z3V27xxw&t=330s> (Example from daily life)



Teaching Colours

Fractions

$\frac{1}{2}$			
$\frac{1}{3}$			
$\frac{1}{4}$			



Story Problem/Word Problem

- Make story problems fun for children, take examples from daily life, things children are familiar with. Use concrete material to explain and solve the problem.
- Read the problem step by step and write the important points for children to see the relevance, discuss the solution, brain storm to find the solution.
- A child need to understand that they need to “decode” the problem/story by applying different operations like addition, subtraction, multiplication or division as per given in the problem.
- A child struggle with such problem because they lack reading comprehension.

Level One

Addition - Sum to 10

3

There are 2 boys and 3 girls in the class. How many kids are there altogether?



1-04

Level One

Addition - Sum to 10

4

Susan picked 3 yellow flowers and 1 red flower. How many flowers did Susan pick in all?



1-04

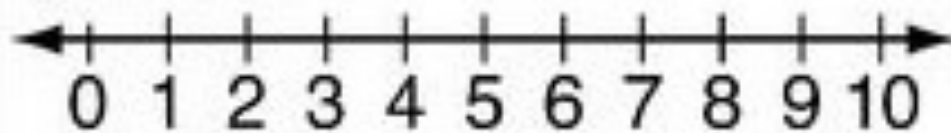
Tad's Bug Jar



Tad put 5 ladybugs in his bug Jar. Tad's sister put 3 more ladybugs in Tad's jar. How many bugs does Tad have in all?

My Picture:

My Number Line:



My Equation:

$$\underline{\quad} + \underline{\quad} = \underline{\quad}$$

My Answer:

 $\underline{\quad}$ bugs

You tube:

- ✓ <https://www.youtube.com/watch?v=a-gDTfRP4I> (math Concepts)

Meaningful Math Activities in Pre-K: Part 2 (Video [#180](#)) (Teaching numbers)

- ✓ <https://www.youtube.com/watch?v=XLfLHXfWrcY> (indoor math activities)

Reading materials:

- ✓ https://resourcecentre.savethechildren.net/node/10017/pdf/elm_brief_final.pdf
- ✓ <https://za.pinterest.com/weber4202/emergent-math/> (Pictures for ideas)
- ✓ <https://za.pinterest.com/jan6921/emergent-mathematics/> (Pictures for ideas)
- ✓ <http://www.pixiehill.org/emergent-math>
- ✓ <http://www.getreadytoread.org/early-learning-childhood-basics/early-math/preschool-math-grows-up-tips-for-teachers>